KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

FIRST PRE BOARD EXAMINATION

MATHEMATICS **MM: 100**

CLASS: XII **TIME: 3 hrs**

General Instructions:

(i) All Questions are compulsory,

(ii) The question paper consists of 29 questions divided into four sections A,B,C and D. Section Acomprises of 4 questions of **one** mark each; Section B comprises of 8 questions of **two** marks each; Sections C comprises of 11 questions of **four** marks each and Section D comprises of 6 questions of **six** marks each.

(iii) There is no overall choice. However, internal choice has been provided in 3 questions of **four** marks each and 3 questions of **six**marks each. You have to attempt only one of the alternatives in all such questions.

(iv) Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.

SECTION - A

1. For what value of k, the matrix $\begin{bmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{bmatrix}$ is skew symmetric?

2. Find the identity element in the set $Q^+$of all positive rational numbers for the operation $*$ defined by $a * b = \frac{3ab}{2}$ for all a, b ∈ Q^+.

3. Find a vector in the direction of vector $-2\hat{\imath} + \hat{\jmath} + 2\hat{k}$ which has magnitude 9 units.

4. Find the principal value of $\cos^{-1}(-\frac{\sqrt{3}}{2})$.

SECTION -B

5. Form differential equation of family of curves

   $y = a\cos(x + b)$ where a and b are arbitrary constants.

6. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find k so that $A^2 - kA + 2I = O$

7. Show that the function f given by $f(x)=x^3 - 3x^2 + 4x$, $x \in \boldsymbol{R}$

   is strictly increasing on **R**.

8. If $e^y(x + 1) = 1$, show that $\frac{dy}{dx} = -e^y$

9. Write the sum of intercepts cut off by the plane $\vec{r}.(2\hat{\imath} + \hat{\jmath} - \hat{k}) - 5 = 0$

10. A die, whose faces are marked 1,2,3 in red and 4,5,6 in green , is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Are A and B independent events.

11. Evaluate $\int \frac{xe^x}{(1+x)^2}dx$

12. Find the value of k if the function defined by

$f(x) = \begin{cases} kx+1, x \leq 5 \\ 3x-5, \; x > 5 \end{cases}$ is continuous at x= 5.

SECTION - C

13. Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$.

14. Using the properties of determinants prove the following:

$$\begin{vmatrix} x+y+2z & x & y \\ z & 2x+y+z & y \\ z & x & x+2y+z \end{vmatrix} = 2(x+y+z)^3$$

15. If A = **R** −{3} and B = **R** −{1}. Consider the function f : A →B defined by $f(x) = \frac{x-2}{x-3}$, for all x ∈ A . Then show that f is bijective.

16. If $y = (\tan^{-1}x)^2$, show that $(1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$

17. Find the intervals in which the function

$f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is i) increasing ii) decreasing.

OR

Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis

18. Evaluate $\int \frac{x^2}{(x^2+9)(x^2+4)}dx$

OR

Evaluate $\int (x-3)\sqrt{x^2+4x+3}\, dx$

19. Solve the differential Equation: $\cos^2 x \frac{dy}{dx} + y = \tan x$; $(0 \leq x < \frac{\pi}{2})$

OR

Solve the differential equation : $xdy - ydx = \sqrt{x^2+y^2}dx$

20. Find the value of λ ,if four points with position vectors $3\hat{\imath} + 6\,\hat{\jmath} \;+\; 9\,\hat{k},\;\; \hat{\imath} + 2\,\hat{\jmath} +\; 3\,\hat{k},\;\; 2\,\hat{\imath} + 3\,\hat{\jmath} +\; \hat{k}$ and $4\,\hat{\imath} + 6\,\hat{\jmath} + \lambda\hat{k}$ are coplanar .

21. Find the shortest distance between the lines whose vector equations are

$$\vec{r} = (1 - 2t)\hat{\imath} + (1 - t)\hat{\jmath} + (t)\hat{k} \quad \text{and}$$
$$\vec{r} = (2 + 3s)\hat{\imath} + (1 - 5s)\hat{\jmath} + (2s - 1)\hat{k}$$

22. There are three coins. One is a two- headed coin (having head on both faces) , another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

23. On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

SECTION - D

24. Using matrices, solve the following system of linear equations

x + 2y – 3z = -4

2x + 3y + 2z = 2

3x – 3y -4z = 11

OR

Using elementary transformations, find the inverse of the following matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

25. Find $\int \left(\log(\log x) + \frac{1}{(\log x)^2} \right) dx$

OR

Evaluate: $\int_1^4 \left(|x - 1| + |x - 2| + |x - 3| \right) dx$.

26. Using integration, find the area of the triangle ABC with vertices as $A(-1, 0)$, $B(1, 3)$ and $C(3, 2)$.

OR

Sketch the region enclosed between the circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$. Using integration, find the area of this enclosed region.

27. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one third of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 tan^2\alpha$.

28. Find the equation of the plane through the line of intersection of the planes x+y+z=1 and 2x+3y+4z =5 which is perpendicular to the plane x – y+z =0. Also, find the distance of the plane obtained above, from the origin

29. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods $F_1$and $F_2$are available. Food F_1 cost Rs 4 per unit and F_2 costs Rs 6 per unit .One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

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