KENDRIYA VIDYALAYA SANGATHAN ERNAKULAM REGION

SECOND PRE BOARD EXAMINATION 2018-19

Class – XII | Time: 3.00Hrs

Subject - Mathematics | Maximum Marks – 100

General instructions

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided in to four sections viz. A, B,C and D. Section A comprises of 4 questions of 1 mark each, Section B comprises of 08 questions of 2 marks each, Section C comprises of 11 questions of 4 marks each and Section D comprises of 6 questions of 6 marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 1 question of 1 mark,3 questions of 2 marks,3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

SECTION A

1 **If f, g : R →R be given by f(x) = $8x^3$ and g $(x) = x^{\frac{1}{3}}$, find (g o f) x**

2 **For what value of k, the matrix** $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ **is not invertible?**

3 If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 5\hat{k}$,find a unit vector in the direction of $\vec{a} - \vec{b}$.

OR

Find the direction cosines of a line which makes equal angles with coordinate axes.

4 **Let * be a binary operation on the set of integers defined by a*b =3a+4b-2, find 4*5.**

SECTION B

5 Prove that $tan^{-1}(\sqrt{x}) = \frac{1}{2}\cos^{-1}[\frac{1-x}{1+x}]$

6 Show that the matrix B'AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

7 Solve for x.

$(tan^{-1}x)^2 + (cot^{-1}x)^2 = \frac{5\pi^2}{8}$

8 Verify mean value theorem for the function f(x) = $x^3 - 5x^2$ -3x in the interval [1,3].

OR

Verify Rolle's theorem for the function f(x)= Sin2x in $\left[0, \frac{\pi}{2}\right]$

9 Evaluate $\int \frac{dx}{1-tanx}$

OR

Evaluate $\int \frac{(x^2+sin^2x)sec^2\ x}{1+x^2}$ dx

10 Form the differential equation of family of circles touching the x axis at the origin

OR

Form the differential equation of family of ellipses having foci on y axis and centre at the origin.

11 Prove that the vectors $\vec{a}$,$\vec{b}$and $\vec{c}$, are coplanar if $\vec{a}+\vec{b}$,$\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar

12 A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?

SECTION C

13 Using properties of determinants, Show that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

14 Differentiate the function f(x) = x^{sinx} + $sin(x^x)$ with respect to x.

OR

If y= log(1+ $t^2 + t^4$), x = tan^{-1} t , find $\frac{d^2y}{dx^2}$

15 If y = $\left[x + \sqrt{x^2-1}\right]^m$, then show that ($x^2$ - 1) $\frac{d^2y}{dx^2}$ + x $\frac{dy}{dx}$ - m^2y = 0

16 **Find the intervals in which the function f(x) = $1 - 12x - 9x^2 - 2x^3$ is increasing or decreasing.**

OR

Find the equation of the normal line to the curve $y(x-2)(x-3) - x + 7 = 0$ at the point where it meets the x −axis.

17 **Let N be the set of all natural numbers and let R be a relation on NXN ,defined by (a,b)R(c,d) <=> ad (b+c) = bc (a+d) for all (a,b),(c,d) ∈ NXN .Prove that R is an equivalence relation.**

18 **Evaluate** $\int e^x \frac{\left(x^2+1\right)}{\left(x+1\right)^2} dx$

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19 **Solve the differential equation $\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x$, given that $y = 0$ when $x = \frac{\pi}{2}$**

OR

Solve the differential equation $(x\,dy - y\,dx)y\sin\frac{y}{x} = (y\,dx + x\,dy)x\cos\frac{y}{x}$ given that y=π when x=3

20 Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d} = 15$.

21 A biased die is twice as likely to show an even number as an odd number. The die is rolled three times. If occurrence of an even number is considered as a success, then write the probability distribution of number of successes. Also find the mean number of successes.

22 **Two groups are competing for the position on the board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.**

23 Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3).

SECTION D

24 **Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}(1/3)$.**

OR

A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible?

25 **Find the product** $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$**.Using the product solve the system of equations**

x-y+2z = 1, 2y-3z = 1 , 3x-2y+4z = 2.

OR

If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ **find** A^{-1}**.Using** A^{-1}**solve the system of equations**

2x-3y+5z = 11, 3x+2y -4z=-5, x+y-2z = -3.

26 **Using integration, find the area of the region** $\{(x, y): |x - 1| \leq y \leq \sqrt{5 - x^2}\}$.

OR

Find the area of the region enclosed between the circles $x^2 + y^2 = 9$ **and** $(x - 3)^2 + y^2 = 9$ **using integration.**

27 **Evaluate by using properties of definite integrals** $\int_0^{\pi} \frac{xdx}{a^2cos^2x+b^2sin^2 x}$

28 **Find the equation of the plane which contains the point (1,-1, 2) and is perpendicular to both the planes 2x+3y-2z=5 and x+2y-3z=8. Hence find the distance of the point P(-2,5,5) from the plane obtained above.**

29 **A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs.5 and that from a shade is Rs.3. assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximize his profit?**

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